Cryptanalysis of the FLIP Family of Stream Ciphers

FSE 2016 rump session

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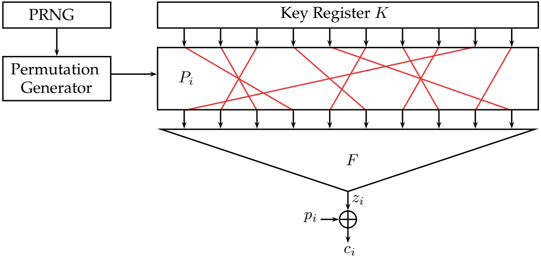
The FLIP Family of Stream Ciphers

Pierrick Méaux, Anthony Journault, François-Xavier Standaert and Claude Carlet,
Towards Stream Ciphers for Efficient FHE with Low-Noise Ciphertexts,
EUROCRYPT 2016.



- **Constant** register storing key K (N bits),
- Permutation generator,
- Filtering function F

The Filtering Function F

$$F\ (x_0, .., x_{n_1-1}, x_{n_1}, x_{n_1+1}, .., x_{n_1+n_2-2}, x_{n_1+n_2-1}, x_{n_1+n_2}, x_{n_1+n_2+1}, x_{n_1+n_2+2}, .., x_{n_1+n_2+n_3-k}, .., x_{n_1+n_2+n_3-1})$$

$$\begin{aligned} &= x_0 + \ldots + x_{n_1-1} \\ &+ x_{n_1}x_{n_1+1} + x_{n_1+2}x_{n_1+3} + \ldots + x_{n_1+n_2-2}x_{n_1+n_2-1} \\ &+ x_{n_1+n_2} + x_{n_1+n_2+1}x_{n_1+n_2+2} + \ldots + x_{n_1+n_2+n_3-k} \cdots x_{n_1+n_2+n_3-1} \end{aligned}$$

Preliminary version:

FLIP (n_1, n_2, n_3)	key $(N = n_1 + n_2 + n_3)$	Security	degree (k)
FLIP (47, 40, 105)	192	80	14
FLIP (87, 82, 231)	400	128	21

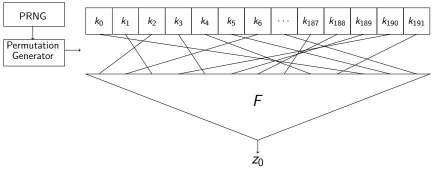
Our Attack

- Known plaintext scenario

Guess-and-determine technique exploiting:

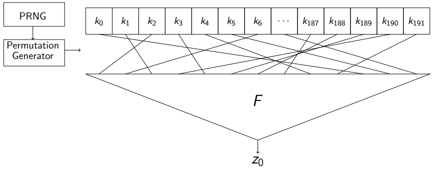
- The **constant** key register
- The **low number** of monomials of degree ≥ 3 in F: $k - 2$

Our Attack



$$z_0 =$$
$$k_{P_0^{-1}(0)} + k_{P_0^{-1}(1)} + k_{P_0^{-1}(2)} + \ldots + k_{P_0^{-1}(45)} + k_{P_0^{-1}(46)} +$$
$$k_{P_0^{-1}(47)} k_{P_0^{-1}(48)} + k_{P_0^{-1}(49)} k_{P_0^{-1}(50)} + \ldots + k_{P_0^{-1}(83)} k_{P_0^{-1}(84)} + k_{P_0^{-1}(85)} k_{P_0^{-1}(86)} +$$
$$k_{P_0^{-1}(87)} + k_{P_0^{-1}(88)} k_{P_0^{-1}(89)} + k_{P_0^{-1}(90)} k_{P_0^{-1}(91)} k_{P_0^{-1}(92)} + \ldots + k_{P_0^{-1}(178)} \cdots k_{P_0^{-1}(191)}$$

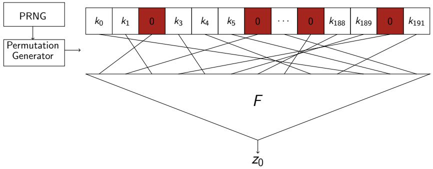
Our Attack



1. Guess $k-2$ null key bit positions

$$
\begin{aligned}
z_0 = \;& k_{P_0^{-1}(0)} + k_{P_0^{-1}(1)} + k_{P_0^{-1}(2)} + \ldots + k_{P_0^{-1}(45)} + k_{P_0^{-1}(46)} + \\
& k_{P_0^{-1}(47)} k_{P_0^{-1}(48)} + k_{P_0^{-1}(49)} k_{P_0^{-1}(50)} + \ldots + k_{P_0^{-1}(83)} k_{P_0^{-1}(84)} + k_{P_0^{-1}(85)} k_{P_0^{-1}(86)} + \\
& k_{P_0^{-1}(87)} + k_{P_0^{-1}(88)} k_{P_0^{-1}(89)} + k_{P_0^{-1}(90)} k_{P_0^{-1}(91)} k_{P_0^{-1}(92)} + \ldots + k_{P_0^{-1}(178)} \cdots k_{P_0^{-1}(191)}
\end{aligned}
$$

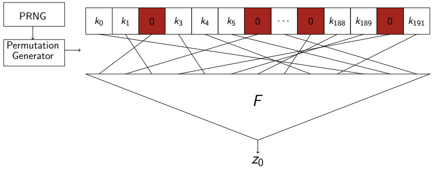
Our Attack



1. Guess $k - 2$ null key bit positions

$$z_0 = k_{P_0^{-1}(0)} + k_{P_0^{-1}(1)} + k_{P_0^{-1}(2)} + \ldots + k_{P_0^{-1}(45)} + k_{P_0^{-1}(46)} + k_{P_0^{-1}(47)} k_{P_0^{-1}(48)} + k_{P_0^{-1}(49)} k_{P_0^{-1}(50)} + \ldots + k_{P_0^{-1}(83)} k_{P_0^{-1}(84)} + k_{P_0^{-1}(85)} k_{P_0^{-1}(86)} + k_{P_0^{-1}(87)} + k_{P_0^{-1}(88)} k_{P_0^{-1}(89)} + k_{P_0^{-1}(90)} k_{P_0^{-1}(91)} k_{P_0^{-1}(92)} + \ldots + k_{P_0^{-1}(178)} \cdots k_{P_0^{-1}(191)}$$

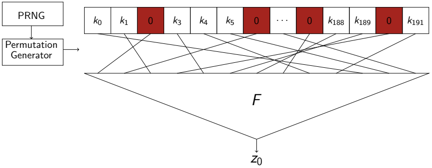
Our Attack



1. Guess $k-2$ null key bit positions
2. Collect quadratic equations

$$z_0 = k_{P_0^{-1}(0)} + k_{P_0^{-1}(1)} + k_{P_0^{-1}(2)} + \dots + k_{P_0^{-1}(45)} + k_{P_0^{-1}(46)} + k_{P_0^{-1}(47)}k_{P_0^{-1}(48)} + k_{P_0^{-1}(49)}k_{P_0^{-1}(50)} + \dots + k_{P_0^{-1}(83)}k_{P_0^{-1}(84)} + k_{P_0^{-1}(85)}k_{P_0^{-1}(86)} + k_{P_0^{-1}(87)} + k_{P_0^{-1}(88)}k_{P_0^{-1}(89)} + k_{P_0^{-1}(90)}k_{P_0^{-1}(91)}k_{P_0^{-1}(92)} + \dots + k_{P_0^{-1}(178)} \cdots k_{P_0^{-1}(191)}$$

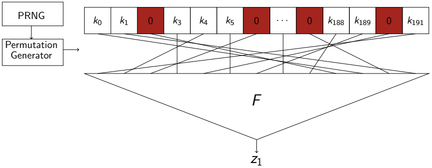
Our Attack



1. Guess $k-2$ null key bit positions
2. Collect quadratic equations

$$z_0 = \cancel{k_{P_0^{-1}(0)}} + \cancel{k_{P_0^{-1}(1)}} + k_{P_0^{-1}(2)} + \dots + k_{P_0^{-1}(45)} + k_{P_0^{-1}(46)} +$$
$$\cancel{k_{P_0^{-1}(47)} k_{P_0^{-1}(48)}} + k_{P_0^{-1}(49)} k_{P_0^{-1}(50)} + \dots + k_{P_0^{-1}(83)} k_{P_0^{-1}(84)} + k_{P_0^{-1}(85)} k_{P_0^{-1}(86)} +$$
$$k_{P_0^{-1}(87)} + \cancel{k_{P_0^{-1}(88)} k_{P_0^{-1}(89)}} + k_{P_0^{-1}(90)} k_{P_0^{-1}(91)} k_{P_0^{-1}(92)} + \dots + k_{P_0^{-1}(178)} \cdots k_{P_0^{-1}(191)}$$

Our Attack



1. Guess $k-2$ null key bit positions
2. Collect quadratic equations

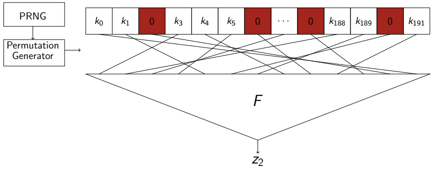
$$z_1 =$$

$$k_{P_1^{-1}(0)} + k_{P_1^{-1}(1)} + k_{P_1^{-1}(2)} + \ldots + k_{P_1^{-1}(45)} + k_{P_1^{-1}(46)} +$$

$$k_{P_1^{-1}(47)}k_{P_1^{-1}(48)} + \cancel{k_{P_1^{-1}(49)}}k_{P_1^{-1}(50)} + \ldots + k_{P_1^{-1}(83)}k_{P_1^{-1}(84)} + \cancel{k_{P_1^{-1}(85)}k_{P_1^{-1}(86)}} +$$

$$\cancel{k_{P_1^{-1}(87)}} + k_{P_1^{-1}(88)}k_{P_1^{-1}(89)} + k_{P_1^{-1}(90)}k_{P_1^{-1}(91)}k_{P_1^{-1}(92)} + \ldots + \cancel{k_{P_1^{-1}(178)} \cdots k_{P_1^{-1}(191)}}$$

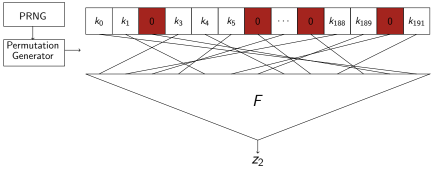
Our Attack



1. Guess $k-2$ null key bit positions
2. Collect quadratic equations

$$z_2 =$$
$$k_{P_2^{-1}(0)} + k_{P_2^{-1}(1)} + k_{P_2^{-1}(2)} + \dots + k_{P_2^{-1}(45)} + k_{P_2^{-1}(46)} +$$
$$k_{P_2^{-1}(47)}k_{P_2^{-1}(48)} + k_{P_2^{-1}(49)}k_{P_2^{-1}(50)} + \dots + k_{P_2^{-1}(83)}k_{P_2^{-1}(84)} + k_{P_2^{-1}(85)}k_{P_2^{-1}(86)} +$$
$$k_{P_2^{-1}(87)} + k_{P_2^{-1}(88)}k_{P_2^{-1}(89)} + \cancel{k_{P_2^{-1}(90)}k_{P_2^{-1}(91)}k_{P_2^{-1}(92)}} + \cancel{\dots} + \cancel{k_{P_2^{-1}(178)} \cdots k_{P_2^{-1}(191)}}$$

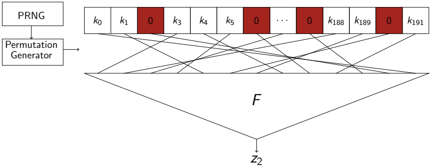
Our Attack



1. Guess $k-2$ null key bit positions
2. Collect quadratic equations
3. Solve the system

$$z_2 = k_{P_2^{-1}(0)} + k_{P_2^{-1}(1)} + k_{P_2^{-1}(2)} + \dots + k_{P_2^{-1}(45)} + k_{P_2^{-1}(46)} +$$
$$k_{P_2^{-1}(47)} k_{P_2^{-1}(48)} + k_{P_2^{-1}(49)} k_{P_2^{-1}(50)} + \dots + k_{P_2^{-1}(83)} k_{P_2^{-1}(84)} + k_{P_2^{-1}(85)} k_{P_2^{-1}(86)} +$$
$$k_{P_2^{-1}(87)} + k_{P_2^{-1}(88)} k_{P_2^{-1}(89)} + \cancel{k_{P_2^{-1}(90)} k_{P_2^{-1}(91)} k_{P_2^{-1}(92)}} + \cancel{\dots} + \cancel{k_{P_2^{-1}(178)} \cdots k_{P_2^{-1}(191)}}$$

Our Attack



1. Guess $k-2$ null key bit positions
2. Collect quadratic equations
3. Solve the system
4. A correct guess gives the key

$$z_2 =$$
$$k_{P_2^{-1}(0)} + k_{P_2^{-1}(1)} + k_{P_2^{-1}(2)} + \ldots + k_{P_2^{-1}(45)} + k_{P_2^{-1}(46)} +$$
$$k_{P_2^{-1}(47)}k_{P_2^{-1}(48)} + k_{P_2^{-1}(49)}k_{P_2^{-1}(50)} + \ldots + k_{P_2^{-1}(83)}k_{P_2^{-1}(84)} + k_{P_2^{-1}(85)}k_{P_2^{-1}(86)} +$$
$$k_{P_2^{-1}(87)} + k_{P_2^{-1}(88)}k_{P_2^{-1}(89)} + \cancel{k_{P_2^{-1}(90)}k_{P_2^{-1}(91)}k_{P_2^{-1}(92)}} + \cancel{\ldots} + \cancel{k_{P_2^{-1}(178)} \cdots k_{P_2^{-1}(191)}}$$

Results

Preliminary version:

degree:	1 ⋯ 1	2	⋯	2	1	2	⋯	k

$$F: \underbrace{x_0 + \ldots + x_{n_1-1}}_{n_1} + \underbrace{x_{n_1}x_{n_1+1} + \ldots + x_{n_1+n_2-2}x_{n_1+n_2-1}}_{n_2} + \underbrace{x_{n_1+n_2} + x_{n_1+n_2+1}x_{n_1+n_2+2} + \ldots + x_{n_1+n_2+n_3-k}\cdots x_{n_1+n_2+n_3-1}}_{n_3}$$

variables: n_1, n_2, n_3

FLIP (n_1, n_2, n_3)	key (N)	Security	degree (k)	C_T	C_D	C_M
FLIP (47, 40, 105)	192	80	14	$2^{54.5}$	$2^{40.3}$	$2^{28.0}$
FLIP (87, 82, 231)	400	128	21	$2^{68.1}$	$2^{58.5}$	$2^{32.3}$

Results

Preliminary version:

degree:	1 ⋯ 1	2 ⋯ 2	1 2 ⋯ k
F:	$x_0 + \dots + x_{n_1-1}$	$+ x_{n_1}x_{n_1+1} + \dots + x_{n_1+n_2-2}x_{n_1+n_2-1}$	$+ x_{n_1+n_2} + x_{n_1+n_2+1}x_{n_1+n_2+2} + \dots + x_{n_1+n_2+n_3-k} \cdots x_{n_1+n_2+n_3-1}$
variables:	n_1	n_2	n_3

FLIP (n_1, n_2, n_3)	key (N)	Security	degree (k)	C_T	C_D	C_M
FLIP (47, 40, 105)	192	80	14	$2^{54.5}$	$2^{40.3}$	$2^{28.0}$
FLIP (87, 82, 231)	400	128	21	$2^{68.1}$	$2^{58.5}$	$2^{32.3}$

New version:

degree: 1 ⋯ 1, 2 ⋯ 2, 1 2 ⋯ k

$$F: x_0 + \dots + x_{n_1-1} + x_{n_1}x_{n_1+1} + \dots + x_{n_1+n_2-2}x_{n_1+n_2-1} + x_{n_1+n_2} + x_{n_1+n_2+1}x_{n_1+n_2+2} + \dots + x_{n_1+n_2-k+\frac{(k+1)k}{2}} \cdots x_{n_1+n_2-1+\frac{(k+1)k}{2}}$$

$$\dots \qquad \dots \qquad \dots$$

$$+ x_{N-\frac{(k+1)k}{2}} + \quad \dots \quad + x_{N-k} \cdots x_{N-1}$$

variables: n_1, n_2; nb (rows)

FLIP $(n_1, n_2, {}^{nb}\Delta^k)$	key (N)	Security	degree (k)
FLIP (42, 128, ${}^{8}\Delta^{9}$)	**530**	80	9
FLIP (82, 224, ${}^{8}\Delta^{16}$)	**1394**	128	16

Thank you!

http://eprint.iacr.org/2016/271.pdf